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A RESEARCH BIBLIOGRAPHY ON "TWO-STAGE" PROBABILISTIC PROGRAMMING 1964-1969

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1 September 1969

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A RESEARCH BIBLIOGRAPHY ON "TWO-STAGE" PROBABILISTIC PROGRAMMING 1964-1969

R. Edward Fricks

Technical Memorandum No. 160

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PREFACE

Preliminaries

This presentation is an outgrowth of an earlier report "Probabilistic Programming 1950-1963: An Annotated Bibliography" which has appeared as Technical Memorandum No. 155 of the Department of Operations Research, School of Management, Case Western Reserve University. The organization, format, and rationale for this document is drawn from that of the former which should be consulted as to the structural details of this summary.

It is noted in passing that an attempt has been made to provide the most complete and up-to-date reference citations possible. Where only partial information as to the literature citation is presented in the entry provided, it represents the latest available estimate of the place of appearance of the material discussed as provided by its author. In most instances a copy of the work can be approached through reference to the listed alternate source.

The content of this memorandum arises as a specialization of that of its predecessor document. Whereas the former concern was with the broad area of "probabilistic programming", attention has been restricted here to a mathematical programming problem termed in the literature "two-stage linear programming under uncertainty" (and its varients). Entries [9], [10], [12], [15], [18], [22], [24], [27], [28], [33], [37], [39], [45], [53], [57], [60], [63] of Technical Memorandum No. 155 summarize the development of the theory on this problem in the years 1950-1963; the intent of this paper is to provide a similar service for the years 1964-1969.*

An understanding of this work rests on a working knowledge of the techniques of linear and nonlinear programming and of the rudiments of "chance-constrained", "stochastic", and "two-stage" uncertain linear programs. Since this last is to be the cohesive thought behind the summaries presented, it seems worthwhile to present a brief discussion of the basic "two-stage" formulation used as the point of departure for this survey. For a discussion in greater depth, see the entries cited above or the articles indexed by this bibliography.

The Basic Model

Consider the following analytical construction

$$\text{Minimize } \; cx + E_V\{qY(V)\}$$

$$\text{Subject to: } \; Ax = b$$

$$B(V)x + CY(V) = d(V)$$

$$x \geq 0$$

where x is an n_1-vector of variables, Y is an n_2-vector of variables, c and q are n_1- and n_2-vectors of objective function coefficients, respectively, A is an $m_1 \times n_1$ matrix, b is an m_1-vector, B is an $m_2 \times n_1$ matrix, C is $m_2 \times n_2$ matrix, d is an m_2-vector, and V is a random variable assuming values v in accordance with some probability distribution function F_V. In the given formulation attention is called to the fact that B, Y and d are functions of the random variable V.

*Articles of the first index referenced by the papers here reviewed have been cited by the numerical indicator previously adopted, e.g., the list above.

Such problems have come to be called "stochastic programs with recourse" in the literature. One possible further generalization is to allow the matrix C and the vector q to be defined by the random variable also. If C remains "fixed" the problem is one of solving a "stochastic program with fixed recourse."

An interpretation of this mathematical model places in evidence the two-stage nature of the associated decision process. One is faced with a problem of decision making in the face of a set of uncertain future states of nature. Circumstances are that some plan (a decision x) must be adopted immediately subject to the situational constraints $(Ax = b$ and $x \geq 0)$ but an adjustment decision (Y) may be adopted later if compensation for unforeseen circumstances (V) is required for successful closure of the anticipated program $(CY = d(V) - B(V)x)$. The program adopted is to be evaluated relative to the sum of the cost of the initial ("first-stage") decision (cx) and the resulting expected cost of the later ("second-stage") decision $E_V\{qY(V)\}$.

A simple example of this problem is a two-stage production planning process in the face of uncertain demands. Production is undertaken in the first period as reflected by the technology matrix A and the quantity of goods produced b and the costs of production c. Demands occur in the second period in accordance with a given distribution with any discrepancies between first-stage usable production $B(V)x$ and the realized demands $d(V)$ being resolved through recourse to a compensatory decision $Y(V)$ with associated costs q.

Acknowledgment

The assistance of Elsie Finley and Carrol Gensert of Sears Library, Case Western Reserve University in accumulating the reference materials reviewed in this report is gratefully acknowledged. My sincere appreciation also goes to Susanne Preston of the Department of Operations Research for correcting my spelling errors, adding commas, and typing this manuscript.

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SELECTED ABSTRACTS ON "TWO-STAGE" PROBABILISTIC PROGRAMMING
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[1A] O. L. Mangasarian, "Nonlinear Programming Problems with Stochastic Objective Functions," Management Science, Vol. [illegible], No. 2, January, 1964, pp. 353-359.

Nonlinear programming problems in which the coefficients of the arguments of the objective function are subject to stochastic variation are the subject of this paper. It is shown that the expected return obtained by observing the random coefficients and then making a decision is greater than the expected return defined by solving the associated two-stage problem. This latter return is demonstrated to be greater than the return expected if the decision found by solving the deterministic program formed by replacing the stochastic coefficients by their expectations is implemented. This last again is found to be greater than the return obtained by solving the deterministic program formed by replacing the stochastic coefficients by their expectations. A calculable upper bound on the return expected from the two-stage problem is stated which is applicable given that the objective is convex in the stochastically varying coefficients of its arguments.

References: [25], [27], [2A]

Alternates:

Referenced by: [63], [17A], [28A], [32A], [44A], [47A]

[2A] O. L. Mangasarian and J. B. Rosen, "Inequalities for Stochastic Nonlinear Programming Problems," Operations Research, Vol. 12, No. 1, January-February, 1964, pp. 143-154.

A two-stage stochastic nonlinear programming problem is formed from a two-stage linear program by replacing each linear functional in the constraint set of the former by a concave function and each linear functional in the objective by a convex function. This paper establishes several propositions for this class of problems. It is shown that the objective form is a convex function of the right-hand-side coefficients and that it is a convex function of the first stage decision variables for any fixed value of the right-hand-side coefficients. The main result states that the expected cost obtained by implementing the decision defined by solving the problem obtained by replacing the stochastic right-hand-side coefficients by their expectations is greater than the expected cost obtained by solving the two-stage problem. The latter is shown to be greater than the expected cost which would obtain if observation of the stochastic right-hand-side coefficients were made prior to making the "first-stage" decision. This last, in turn, is shown to be greater than the cost which would result from the solution of the deterministic program in which the right-hand-side coefficients are replaced by their expectations. An upper and lower bound to the two-stage stochastic problem objective is given in terms of the solutions for two deterministic problems of the same size.

References: [9], [10], [25], [27].

Alternates: O. L. Mangasarian and J. B. Rosen, "Inequalities for Stochastic Nonlinear Programming Problems," Recent Advances in Mathematical Programming, R. L. Graves and P. Wolfe, eds., McGraw-Hill, New York, 1963, p. 123 (Abstract). Also under the same authors and title as Report P 1127, Shell Oil Development Company, Emeryville, California.

Referenced by: [1A], [3A], [16A], [17A], [27A], [28A], [32A], [38A], [47A].

[3A] Bui Trong Lieu, "On a Problem of Convexity and its Application to Nonlinear Stochastic Programming," _Journal of Mathematical Analysis and Applications_, Vol. 8, No. 2, April, 1964, pp. 177-187.

This paper in one sense is a review and restatement of results contained in the references cited. It, at the same time, is an extension of the same results by virtue of the mathematical preciseness of the presentation which introduces a deeper level of abstraction into the discussion. This same depth of abstraction renders the discussion difficult to follow at a first reading.

Two subdivisions, one covering a deterministic description and the other a random model, organize the results. The presentation is such that the deterministic results carry through quite nicely to the random characterization. The usual inequalities applicable to the objective form of a two-stage uncertain nonlinear program are developed on the basis of given convexity assumptions and proofs.

References: [27], [37], [45], [53], [2A].

Alternates:

Referenced by: [16A], [32A].

[4A] David Köhler and Roger Wets, "Programming Under Uncertainty: An Experimental Code for the 'Complete' Problem," Operations Research Center, University of California, Berkeley, ORC 64-15, July, 1964.

The topic of this report is, as its title suggests, the presentation of a computer code for a special form of the two-stage linear programming under uncertainty problem. In particular, it is assumed that all decision vectors feasible for the first-stage constraint set will be feasible for the second-stage problem with descrepancies by definition equaling the second-stage decision variable. The algorithm outlined assumes a continuously distributed right-hand-side vector for the second-stage constraint set and uses successive supporting hyperplanes to the objective form to define directions of descent which yield a sequence of solutions converging to the optimum.

A brief statement of the problem is given with the theoretical justification of the equations used in the algorithm left to the cited reference. Two general block diagrams indicative of the program structure accompany the user write up presented. In the latter, sufficient detail is given so that a person having a source deck for the program could successfully run some sample problems. The first pages of output from one test problem which was investigated is included to illustrate the data format, but no source deck listing is in evidence. It should be noted that the program as developed was written in FORTRAN IV and is compatible with the IBSYS Monitor for the IBM 7000 series computers.

References: [15], [11A].

Alternates: David Köhler and Roger Wets, "Programming Under Uncertainty: An Experimental Code for the Complete Problem," Boeing Scientific Research Laboratories, Document D1-82-0391, (Mathematics Research Laboratories, Mathematical Note No. 378), December, 1964.

Referenced by: [11A], [27A], [44A].

[5A] Wlodzimierz Szwarc, "The Transportation Problem with Stochastic Demand," Management Science, Vol. 11, No. 1, September, 1964, pp. 33-50.

The transportation problems considered in this paper are assumed to have consumer requirements which are described by random variables having bounded supports for their marginal distribution functions. It is assumed that the distribution functions can be approximated satisfactorily by step functions, and that the appropriate objective is to minimize the cost of allocating supplies to outlets plus the sum of the expected (piece-wise linear) cost due to undersupply or oversupply at each destination. An approximating deterministic linear program is developed through use of a piece-wise linear approximation to the nonlinear objective form. It incorporates auxiliary variables with upper bounds. A solution procedure is presented and used to solve an example problem.

References: [15], [50], [8A].

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Referenced by:

[6A] G. Hadley, "Stochastic Programming," Nonlinear and Dynamic Programming, Chapter 5, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1964, pp. 158-184.

This chapter presents an operational discussion of stochastic programming problems. That is, several examples are discussed in order to illustrate the formulation procedure for such problems and to point out the difficulties which complicate their solutions. In the main the discussion centers on problems of the "two-stage" variety although "chance-constrained" and "stochastic" formulations are touched upon.

"Nonsequential decision" stochastic programming problems and "sequential decision" stochastic programming problems is the basic dichotomy adopted. Fundamentally, this categorization corresponds to considering "two-stage" and "multi-stage" linear programming under uncertainty problems. Under the first category an inventory problem, a "knapsack" problem, and three allocation-type problems (a transportation problem, a problem of allocation of aircraft to routes, and a production scheduling problem) are formulated and reduced to their equivalent convex programs. In each case random variables appear only in the right-hand-side requirements vector. A solution procedure is proposed which uses a polygonal approximation to the convex objective and allows use of the decomposition algorithm for linear programs to achieve a solution. An n-period inventory problem is discussed at length to exemplify a sequential decision stochastic programming problem. It is acknowledged that in the main such problems are difficult to solve. Dynamic programming is suggested as a solution technique applicable for the solution of a certain subclass of this type of problem.

"Two-stage" stochastic problems with random variables appearing in the technological coefficients, the "expected cost of uncertainty", the propriety of replacing the random variables by their expectations, and the problem of determining the a posteriori distribution of the optimum of a stochastic linear program a priori are all taken up briefly to round out the discussion.

References: [5], [9], [10], [11], [14], [15], [18], [19], [20], [26], [27], [29], [31], [36].

Alternates:

Referenced by: [38A]

[7A] M. Iosifescu and R. Theodorescu, "Linear Programming Under Uncertainty," Colloquium on Applications of Mathematics to Economics, Andras Prékopa, ed., Akadémiai Kiadó, Budapest, 1965, pp. 133-139.

This paper is a survey of results on stochastic linear programming placed in a decision making context. Results from several previous papers by the same authors are quoted as a part of the review. Some question arises as to the appropriateness of including this paper in a bibliography on "two-stage" programming as the approach adopted is more consistent with the philosophy of "stochastic programming." Perhaps the best justification in this regard (other than the fact that the approach adopted is "interesting") is that the discussion considers uncertain linear programs in a two-person game theoretic sense. Incorporation of the "payoff matrix" through use of a "loss function" into the linear objective form (as is done) will permit the problems discussed to be cast in the form of two-stage linear programming under uncertainty problems (which is not done).

This paper opens with some observations on the solution sets for stochastic linear programs. One comment on the containment of one solution set within another has not appeared elsewhere. The next section discusses linear programs under combinations of "risk" and "uncertainty" placing emphasis on the aforementioned game theoretic interpretation. The latter portion of the discussion relates the subject formulation to the "statistical decision theory" approach to decision making under uncertainty. The abstract probabilistic structure introduced to characterize the situations presented is well thought out and is one of the better definitions which have appeared.

References: [45], [52].

Alternates: This entry was located by reference to a preliminary version of a "Bibliography on Programming Under Risk," supplied by Mr. Douglas V. Smith, Harvard University Center for Population Studies, 22 Plympton Street, Cambridge, Massachusetts 02138. [52] is the closest equivalent to the tenth reference of this paper that has been abstracted in this series to date.

Referenced by:

[8A] A. Charnes, W. W. Cooper and G. L Thompson, "Constrained Generalized Medians and Hypermedians as Deterministic Equivalents for Two-Stage Linear Programs Under Uncertainty," Management Science, Vol. 12, No. 1, September, 1965, pp. 83-112.

This paper describes an equivalent formulation of the usual statement of the two-stage linear programming under uncertainty problem in which the second-stage constraints are incorporated into the program objective functional. This is made possible by two observations. First, it is observed that the second-stage constraints are definitional in nature in that they only serve to define the nature of the decision variables to be included in the objective functional. Second, it is noted that the second-stage problem solution as a function of the first-stage decision variable and the stochastic right-hand-side vector can be used to replace the second-stage decision variables appearing in the objective functional. The result is a completely equivalent formulation involving the first-stage constraints, the first-stage decision variables, and an objective functional incorporating the first absolute moments of the distribution of the right-hand-side vector with respect to the first-stage decision variables. These last, in view of the property of distribution functions that the first absolute moment becomes a minimum when the moment point is the median of the distribution, can be used to simplify the optimization required with respect to the first stage decision variables. Numerous examples are given at each step of the development which motivate the theory presented and make this an exceptionally readible paper.

References: [5], [7], [8], [9], [10], [12], [14], [15], [18], [19], [20], [23], [24], [26], [27], [28], [29], [30], [31], [33], [37], [38], [39], [41], [42], [43], [50], [53], [55].

Alternates: A. Charnes, W. W. Cooper, and G. L. Thompson, "Constrained Generalized Medians and Linear Programming Under Uncertainty," ONR Research Memorandum No. 9, July 6, 1961, Evanston, Ill., Northwestern University, The Technological Institute - ONR Research Project on Temporal Planning and Management Decision Under Risk and Uncertainty and Pittsburgh, Carnegie Institute of Technology, Graduate School of Industrial Administration - ONR Research Project on Planning and Control of Industrial Operations. Also by the same authors and under the same title as ONR Research Memorandum No. 41, July 6, 1961, and again as "Chance-Constrained Studies for Linear Programming Under Uncertainty, Part One: Constrained Generalized Medians and Two-Stage Problems with General Linear Structure," ONR Research Memorandum No. 106, December, 1962, and as "Chance Constrained Studies for Linear Programming Under Uncertainty, I.," ONR Research Memorandum 120, July 1964.

Referenced by: [50], [58], [5A], [22A], [25A], [26A], [27A], [29A], [30A], [31A], [37A], [46A].

[9A] A. C. Williams, "On Stochastic Linear Programming," Journal of the Society of Industrial and Applied Mathematics, Vol. 13, No. 4, December, 1965, pp. 927-940.

Linear programs in which the inhomogeneous term (the right hand side) of the constraining relations are random variables and for which the objective functional is the expectation of the usual linear form augmented by a nonlinear term (which incorporates consideration of discrepancies between programmed supply and demand) are discussed. The problem covered is therefore a special case of the "two-stage linear programming under uncertainty" formulation for stochastic linear programs.

Several Theorems and Lemmas are presented which define necessary and sufficient conditions for program feasibility and characterize solutions for the subclass of two-stage problems defined by the assumptions of the subject formulation. The saddle point conditions of Kuhn and Tucker and the theory of duality for linear programs provide the foundation for the analysis. The treatment given is theoretical and quite comprehensive for the problem type considered.

References: [24], [28], [37], [45], [57], [60], [26A].

Alternates: A. C. Williams, "Errata: On Stochastic Linear Programming," Journal of the Society of Industrial and Applied Mathematics, Vol. 15, No. 1, January, 1967, p. 228, makes one small correction to this article. This was also pointed out in A. Charnes, M. J. L. Kirby, and W. M. Raike, "On a Special Class of Constrained Generalized Median Problems," The Technological Institute, Northwestern University, Evanston, Ill., Systems Research Memorandum No. 155, April, 1966.

Referenced by: [12A], [13A], [20A], [21A], [24A], [25A], [26A], [28A], [29A], [38A], [46A], [47A].

[10A] R. J.-B. Wets, "Programming Under Uncertainty: The Equivalent Convex Program," Journal of the Society of Industrial and Applied Mathematics, Vol. 14, No. 1, January, 1966, pp. 89-105.

This paper considers the general two-stage linear program under uncertainty problem. The feasible solution sets for the first and second stage problems are shown to be convex. Farka's Lemma is applied to produce a feasibility test for a given trial solution which apparently could be used as the basis of a solution algorithm by virtue of the fact that it reduces the set of feasible solutions which must be searched for the optimum. An equivalent convex program for the two-stage uncertain linear program is developed and several propositions detailing its properties are discussed. Necessary and sufficient conditions for optimality of a given solution are also described.

Examples in the form of special cases of the general two-stage problem are interspersed throughout the text. These are used to illustrate the main propositions and suggest solution procedures. A mathematical appendix contains a statement of the "well-known" propositions of linear and convex programming which need be invoked for the proofs of the paper.

References: [37], [11A].

Alternates: Roger Jean-Baptiste Robert Wets, "Programming Under Uncertainty," Ph.D. Thesis in Engineering Science, University of California, Berkeley, 1965. Also, Roger Wets, "Programming Under Uncertainty: The Equivalent Convex Program," Boeing Scientific Research Laboratories, Document D1-82-0411 (Mathematics Research Laboratory, Mathematical Note No. 392), February, 1965.

Referenced by: [12A], [15A], [18A], [19A], [20A], [22A], [24A], [25A], [26A], [27A], [28A], [29A], [31A], [32A], [34A], [35A], [36A], [42A], [45A], [46A].

[11A] Roger Wets, "Programming Under Uncertainty: The Complete Problem," Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete, Vol. 4, No. 4, February, 1966, pp. 316-339.

The subject of this paper is a special form of the two-stage linear program under uncertainty in which any discrepancies resulting in a second stage decision problem are resolved by simply adding or subtracting resource units as necessary, i.e., every decision satisfying the "fixed" first-stage constraints is automatically feasible for the second-stage problem. In particular, the form of the problem is such that the second-stage transformation matrix is always a unit matrix.

The first section of the report demonstrates the existence of an equivalent separable convex program for the underlying stochastic program. The properties of the constituent functions comprising the separable objective form are discussed in some detail. Two separable convex programming algorithms are suggested as applicable solution techniques.

The second section discusses special instances by assuming particular distribution functions for the program random variables. It is shown that for finitely distributed right-hand-side random variables the equivalent convex program becomes a linear program with upper bounds on the decision variables. For a uniform distribution a quadratic program is established; for an exponential distribution an approximating convex program is given; and for continuous distribution functions an approximating convex program based upon a series of uniform distributions is suggested.

The third section suggests a solution algorithm for the complete problem in the case of a continuously distributed right-hand-side coefficient random variable. The basis of the algorithm is the solution of a series of linear programs incorporating consideration of approximating hyperplanes to the complete problem convex objective functional as objective forms to generate successive trial solutions.

References: [15], [24], [39], [63], [4A], [18A].

Alternates: Roger Jean-Baptiste Robert Wets, "Programming Under Uncertainty," Ph.D. Thesis in Engineering Science, University of California, Berkeley, 1965. Also, Roger Wets, "Programming Under Uncertainty: The Complete Problem," Boeing Scientific Research Laboratories, Document D1-82-0379 (Mathematics Research Laboratory, Mathematical Note No. 370), October, 1964.

Referenced by: [4A], [10A], [12A], [18A], [19A], [20A], [21A], [22A], [25A], [26A], [27A], [28A], [29A], [30A], [34A], [36A], [42A], [44A], [45A], [46A], [47A].

[12A] Richard Van Slyke and Roger Wets, "Programming Under Uncertainty and Stochastic Optimal Control," Journal of the Society of Industrial and Applied Mathematics on Control, Vol. 4, No. 1, February, 1966, pp. 179-193.

The first two sections of this paper are taken up with a restatement of the two-stage linear programming under uncertainty problem in e abstract mathematical structure which relates to the stochastic optimal c problem formulation. The third section first considers a theory of duality for the abstract structure developed to describe the uncertain programming problem. The statements and proofs are reminiscent of those characterizing the standard deterministic linear programming duality formulation. For the most part, the proofs are somewhat intuitively presented. In the second part of the section it is demonstrated that the duality concept developed is consistent with linear programming duality and an application to a linear control problem discussed. In the latter instance the equivalence of the duality theory derived description and the maximum principle description is indicated.

References: [10], [37], [53], [60], [9A], [10A], [11A].

Alternates: Richard Van Slyke and Roger Wets, "Programming Under Uncertainty and Stochastic Optimal Control," Boeing Scientific Research Laboratories, Document D1-82-0429 (Mathematics Research Laboratory, Mathematical Note No. 405), April, 1965.

Referenced by: [15A], [26A], [28A]

[13A] A. C. Williams, "Approximation Formulas for Stochastic Linear Programming," Journal of the Society of Industrial and Applied Mathematics, Vol. 14, No. 4, July, 1966, pp. 668-677.

The main purpose of this paper is to present a result which limits the error experienced in the value of the objective functional if a special two stage linear program under uncertainty is solved approximately using a certainty equivalent estimate for the right-hand-side random vector. The results presented are valid for any estimate, including replacing the random vector by its expectation. Monitonicity and differentiability properties of the error term described are detailed and a suggestion as to a method for generating successively better estimates is given. A simple example illustrates the nature of the comparisons obtainable between the approximate optimal objective function value and the true optimum.

References: [10], [27], [34], [60], [9A].

Alternates:

Referenced by: [20A], [21A], [25A], [28A], [29A], [38A], [44A], [47A].

[14A] Robert Wilson, "On Programming Under Uncertainty," Operations Research, Vol. 14, No. 4, July-August, 1966, pp. 652-657.

This short paper discusses two-stage uncertain linear programs in which the random right-hand-side vector is normally distributed. The Kuhn-Tucker conditions are used to develop a dual problem from the Lagrangian saddle-point formulation of the problem originally stated. An application of the dual formulation to the derivation of an optimal sampling plan in the case of unknown but *a priori* estimatible means and variances of the marginal distributions of the random right-hand-side vector coefficients is given.

References: [24], [28], [63]

Alternates:

Referenced by: [21A], [29A], [47A]

[15A] Roger Wets, "Programming Under Uncertainty: The Solution Set," Journal of the Society of Industrial and Applied Mathematics, Vol. 14, No. 5, September, 1966, pp. 1143-1151.

This paper is a continuation of an earlier one in which it was shown that the solution set of the general two-stage linear program under uncertainty is a convex set. The main result of this paper is to show that the solution set is not only convex but also polyhedral. It is also shown that the equivalent convex program of a multi-stage linear programming under uncertainty problem (and therefore a fortori for the two-stage equivalent) is that of extremizing a convex function subject to a set of linear constraints. The method of proof is through geometric construction and intuitively quite appealing.

References: [58], [63], [10A], [12A].

Alternates: Roger Wets, "Programming Under Uncertainty: The Solution Set," Boeing Scientific Research Laboratories, Document D1-82-0464 (Mathematics Research Laboratory, Mathematical Note No. 424), August, 1965.

Referenced by: [20A], [23A], [24A], [26A], [27A], [35A], [42A], [46A].

[16A] Bui Trong Lieu, "A Study of Some Inequalities for Nonlinear Stochastic Programming," Nonlinear Programming, Chapter 10, J. Abadie, ed., North-Holland Publishing Company, Amsterdam, and John Wiley & Sons, Inc. (Interscience Publishers Division), New York, 1967, pp. 249-258.

This article is a rewrite and restatement of results presented by the author in two previous papers cited as references. Although some proofs are given, other results are quoted only. The usual inequalities applicable to the objective form of a two-stage uncertain nonlinear program are developed on the basis of suitable convexity and quasi-convexity assumptions and proofs.

References: [27], [37], [45], [53], [2A], [3A].

Alternates:

Referenced by: [32A]

[17A] J. Wessels, "Stochastic Programming," Statistica Neerlandica, Vol. 21, No. 1, 1967, pp. 39-53.

This article is a survey and introduction to stochastic linear programming problems. Although the presentation is quite general, a strong tie to two-stage uncertain linear programming problems is in evidence by virtue of the discussion on loss functions and their relation to the two-stage formulations in the cited references. A game theoretic interpretation in which a decision-maker has to determine a pure strategy given the probability-distribution of strategies of one other player (nature) is used in the presentation. The pay-off function for the first player is assumed to be dependent on the realized value of the criterion and appropriate evaluation of constraint discrepancies. Several alternative evaluation-functions are proposed, discussed, and used as a basis for illustrative examples. These also serve to point up the correspondence which exists between the "two-stage" and "chance-constrained" probabilistic programming problem formulations. An appendix relates the pay-off function of the formulations discussed to the preference function concepts of the economic theory of utility.

References: [9], [26], [27], [28], [37], [43], [45], [50], [54], [55], [62], [63], [1A], [2A].

Alternates:

Referenced by:

[18A] M. El-Agizy, "Two-Stage Programming Under Uncertainty with Discrete Distribution Function," Operations Research, Vol. 15, No. 1, January-February, 1967, pp. 55-70.

Two-stage stochastic linear programs in which the second-stage decision variable constraint matrix can be partitioned into a positive identity matrix and its corresponding negative and in which the stochastic right-hand-side vector has a discrete distribution are discussed in this paper. Equivalent deterministic programs are developed which produce the conclusion that only the "marginal" distributions of the right-hand-side stochastic vector, i.e., the distributions of the individual components, are necessary to the problem solution. Thus complete information as to the distribution of the stochastic right-hand-side vector would be somewhat redundant relative to the application sought.

The deterministic equivalent developed assumes e form of an equivalent linear program with upper bounded decision variables. Optimality conditions are developed which, by virtue of a natural ordering on the relative cost coefficients of the decision variables, could be used to produce a solution algorithm requiring substantially less computational effort than if the standard simplex technique was applied to the problem. No computational results are presented, but three example situations - a stochastic Hitchcock-Koopman's transportation problem, a production-inventory problem under uncert ı demand, and a stochastic Leontief production model - are defined and cast in the form of their corresponding deterministic equivalents.

References: [3], [9], [10], [15], [24], [37], [60], [63], [10A], [11A], [18A].

Alternates: Mostafa El-Agizy, "Programming U--er Uncertainty with Discrete D.F.," Operations Research Center, Ur v sity of California, Berkeley, ORC (-13, 22 July 1964. Also ostafa Mohamed Nabih El-Agizy, "Multistage Programming Under Uncertainty," Ph.D. Thesis, Department of Industrial Engineering, University of California, Berkeley, January, 1965.

Referenced by: [63], [11A], [20A], [28A], [44A], [46A].

[19A] William H. Evers, "A New Model for Stochastic Linear Programming," Management Science, Vol. 13, No. 9, May, 1967, pp. 680-693.

This paper discusses the problem of stochastic linear programming in relation to a specific application. The stochastic problem of charging a metal melting furnace is reduced to a deterministic problem by defining additional terms to be incorporated into the stochastic program objective functional. The terms added are chosen to reflect the application discussed. The deterministic program given is thus a substitute for the usual formulation for a particular application not a theoretical equivalent applicable to the general stochastic programming formulation. The exposition given is explanatory and descriptive rather than theoretically definitive.

References: [5], [8], [9], [10], [14], [15], [18], [19], [24], [26], [27], [28], [30], [37], [43], [45], [53], [54], [55], [60], [10A], [11A].

Alternates: W. Evers, "A New Model for Stochastic Linear Programming," IBM O.R. Report No. FSC67-0017, C.E.S. Rockville, Maryland, August, 1966, 135 pages.

Referenced by: [22A], [25A].

[20A] Shailendra C. Parikh, "Generalized Stochastic Programs with Deterministic Recourse," Operations Research Center, University of California, Berkeley, ORC 67-27, July, 1967.

The model discussed in this paper is an extension of the usual two-stage linear programming under uncertainty structure. Effectively a "third-stage" set of linear constraints are added which incorporate consideration of the deterministic first-stage decision variables and the second-stage decision variables (which depend on the observation of the outcome of some random process). An additional set of decision variables is introduced so as to allow adjustments in the final set of linear relations so as to achieve their equality with the zero vector. The objective of the problem is to select the first-stage decision so as to minimize the cost of making that decision plus the cost expected due to the "forced" second-stage decision and the "compensatory" third-stage decision. One type of problem which may be cast in these terms is that in which a commitment is made in the face of uncertain input availabilities (the first-stage decision), a subsequent action is initiated on the basis of the inputs realized (the "stochastic" second-stage decision), and the commitment is finally realized through such compensatory activities as might be made necessary by the discrepancies found between the inputs predicted at the time of the commitment decision and the inputs realized at the time of the initial commitment activity (the third-stage decision). It is this category of problem that is termed a "generalized stochastic program with deterministic recourse."

Within the above framework some additional specialization is encountered: it is assumed that the second-stage random variable has a discrete distribution with a finite number of points of positive probability.

Two subclasses of this type of programming problem are considered - one in which a feasible solution always exists and the other in which it might not. The former are termed problems with "complete recourse" and the latter problems with "incomplete recourse." In the presentation the method of solution suggested is essentially the same.

After a brief literature review and presentation of the model to be discussed, the main concern of the paper is with the development of a solution algorithm for the class of problems described. Several quoted results lead to the conclusion that a modified cutting plane algorithm might be applicable to the formulation given. The method presented will converge in a finite number of steps to a solution whose value is arbitrarily close to the optimal solution value. A discussion of that approach incorporating convergence "proofs" follows. The length of this dialogue coupled with a complete lack of illustrative examples or numerical results tends to detract from the presentation somewhat.

References: [5], [10], [24], [26], [37], [63], [9A], [10A], [11A], [13A], [18A], [20A], [24A], [42A].

Alternates: Shailendra Chimanlal Parikh, "Generalized Stochastic Programs with Deterministic Recourse," Ph.D. Thesis, Department of Industrial Engineering, University of California, Berkeley, July, 1967.

Referenced by: [27A], [46A].

[21A] A. C. Williams, "Nonlinear Activity Analysis and Duality," Proceedings of the Sixth International Symposium on Mathematical Programming, H. W. Kuhn, ed., Princeton University Press, Princeton, New Jersey,

This paper is both an introduction and a review of duality concepts for convex programming problems. Primal-Dual problems are developed on the basis of a restricted conjugate function space for the primal problem which incorporates special linear functionals. The development as presented does not require differentiability assumptions. A two-stage uncertain linear program which has appeared in previous publications is used to illustrate the duality concept developed. To further enhance intuitive understanding, economic interpretations of the conjugate (dual) forms is given in terms of the interrelationships between a hypothetical entrepreneur and a hypothetical contractor. The entrepreneur is faced with the problem of producing the goods he requires for his activities or purchasing them from the contractor. The contractor must resolve a dual decision problem.

References: [9A], [11A], [13A], [14A], [26A], [29A]

Alternates: A. C. Williams, "Programming Under Uncertainty: The Linear Recourse Problem," paper presented at the Sixth International Symposium on Mathematical Programming, Princeton University, Princeton, New Jersey, August, 1967.

Referenced by: [47A].

[22A] R. Jagannathn, "An Algorithm for Differentiable Convex Functional Programming with Examples and Applications to Linear Programming Under Uncertainty," Management Sciences Research Group, Graduate School of Industrial Administration, Carnegie-Mellon University, Management Sciences Research Report No. 114, August, 1967.

This report is concerned with the presentation of a computational algorithm for solving convex programming problems with linear constraints and a "continuously" differentiable criterion function. The procedure is discussed in Section 3 and makes use of a quadratic approximation to the original program objective form. Given an initial feasible solution for the total problem, a parametric, quadratic objective form is used to produce a revised solution This "quadratic iteration" is continued until a close enough approximation to the optimum is achieved. A simple numerical example is given to illustrate the method's operation.

In Sections 4, 5 and 6 of the report the applicability of the method to two-stage linear programming under uncertainty problems is discussed. It is shown that the necessary conditions assumed by the algorithm are satisfied by stochastic programs with "complete" recourse having right-hand-side vectors with absolutely continuous distribution functions, by programs having both a stochastic right-hand-side vector and a stochastic constraint matrix associated with the first-stage decision variables in the second-stage constraints with the random variables of concern described by a multivariate normal distribution, and by similar programs with stochastic penalty functions which satisfy appropriate convexity assumptions. A single-period portfolio selection problem is formulated to exemplify the practicality of the abstract structures discussed.

References: [5], [9], [10], [12], [20], [24], [26], [30], [37], [39], [42], [45], [50], [53], [63], [8A], [10A], [11A], [19A], [24A], [30A].

Alternates: R. Jagannathn, "An Algorithm for Differentiable Convex Functional Programming with Examples for Applications to Linear Programming Under Uncertainty," paper presented at the Tenth American Meeting of The Institute of Management Sciences, Atlanta, Georgia, October 1-3, 1969. Also Rajagopalan Jagannathn, "Solution Procedures for Multi-Stage Linear Programming Under Uncertainty and Applications," Management Sciences Research Group, Graduate School of Industrial Administration, Carnegie-Mellon University, Management Sciences Research Report No. 158, May, 1969 (Ph.D. Thesis).

Referenced by: [37A]

[23A] David A. Köhler, "Projections of Convex Polyhedral Sets," Operations Research Center, University of California, Berkeley, ORC 67-29, August, 1967.

This paper, which is concerned with the elimination of a subset of a set of decision variables in a group of linear inequalities, has direct application to problems formulable as two-stage linear programming under uncertainty problems. Given a set of linear inequalities in two sets of decision variables, it is shown how an equivalent set of inequalities can be constructed which incorporate only the second set of original decision variables. This is done by considering elements of the null space of the constraint matrix of the first set of decision variables. It is shown that elimination of the first set of decision variables is equivalent to constructing a projection of the convex polyhedral set defined by the total set of inequalities onto the space spanned by the second set of decision variables A method for accomplishing this task (a modification of the Fourier-Motzkin method for eliminating variables from a set of linear inequalities) is discussed. The procedure developed is basically an algorithm for finding the (unique) frame of the pointed convex cone of elements in the null space of the eliminated set of decision variables constraint matrix. The several examples interspersed throughout the text contribute toward making the presentation a most readible one.

References: [63], [15A].

Alternates: Available as AD 659301. Also, David Anthony Köhler, "Projections of Convex Polyhedral Sets," Ph.D. Thesis, Department of Industrial Engineering, University of California, Berkeley, August, 1967.

Referenced by:

[24A] David W. Walkup and Roger J.-B. Wets, "Stochastic Programs with Recourse," Journal of the Society of Industrial and Applied Mathematics, Vol. 15, No. 5, September, 1967, pp. 1299-1314.

Stochastic programs with recourse are a generalization of a particular form of stochastic linear programming called "two-stage linear programs under uncertainty". The former are obtained from the latter by allowing some (or all) of the objective function coefficients and the second-stage constraining relation coefficients to be subject to stochastic fluctuations. This is in contrast to the usual two-stage formulation in which only the right-hand-side coefficients of the second-stage constraining relations are presumed to be described by a random variable.

The tenor of the paper is to establish conditions under which stochastic programs with recourse can be reduced to an equivalent deterministic convex programming problem (continuous distributions of the program random variables are assumed throughout). This is accomplished by first establishing conditions under which the program constraint set is convex and then demonstrating the convexity of the associated program objective form. In the former instance one of the prime items for consideration is the propriety of replacing the domain of definition of the program random variable by its closure and yet maintaining the integrity of the original problem formulation. Conditions under which this procedure will produce a consistent answer relative to the initial problem are established by recourse to some topological set constructions and some continuity properties of convex-cone-valued mappings. It turns out that the second-stage constraint matrix associated with the second-stage decision variables is the pivotal factor in the discussion. In the case of the objective form, convexity is established in the usual manner. It is also shown that, under mildly restrictive conditions, the contribution of the second-stage variables to the program objective is finite and satisfies a Lipschitz condition.

The notation and constructions adopted in this article do not allow its content to be absorbed through a cursory reading. In fact, one wonders if at times conceptual clarity has not been sacrificed for the sake of mathematical abstraction.

References: [9], [10], [37], [57], [9A], [10A], [15A], [25A].

Alternates: David W. Walkup and Roger J. B. Wets, "Stochastic Programs with Recourse," Boeing Scientific Research Laboratories, Document D1-82-0551 (Mathematics Research Laboratory, Mathematical Note No. 474), July, 1966

Referenced by: [20A], [22A], [24A], [26A], [27A], [28A], [30A], [31A], [32A], [36A], [37A], [39A], [40A], [45A], [46A]

[25A] David W. Walkup and Roger J. B. Wets, "Stochastic Programs with Recourse: Special Forms," Boeing Scientific Research Laboratories, Document D1-82-0627 (Mathematics Research Laboratory, Mathematical Note No. 516) August, 1967.

A "stochastic program with recourse" is obtained from a "two stage" linear programming under uncertainty problem by assuming that, in addition to the right-hand-side coefficients of the second-stage constraints, the coefficients of the linear objective form and the second-stage constraint technology matrices are subject to random fluctuations. Thus, in general, an arbitrary first-stage decision may not allow a feasible second-stage decision to be initiated for all possible combinations of the stochastic parameter values. The concern of this document is with some special forms of this class of problems which are theoretically and computationally tractable.

The key entity in the stochastic program with recourse problem is the second-stage technology matrix associated with the second-stage decision variables. If this matrix is such that all first-stage decisions are acceptable to the total problem with respect to feasibility, a stochastic program with complete recourse is the result; if the matrix is a constant, the problem is one of fixed recourse; and if the matrix is square and nonsingular with probability one, stable recourse occurs. The preceding then are the "special forms" discussed by the subject paper.

The first part of the presentation is taken up with definitions and a statement of results drawn from a prior paper which have bearing on the special forms to be discussed. For the "special forms" considered conditions under which the equivalent deterministic problem is a convex program are discussed. Various specializations permit conclusions with respect to the separability, convexity, and existence of bounded first partial derivatives of the program objective forms. This last characteristic leads to some speculation on the appropriateness of previous solution procedures for application to the problem types under discussion. It is also shown how previous results can be accommodated in the framework of stochastic programs with recourse. Examples given include a "stochastic linear program" incorporating the "active" approach to decision making and a conditional probability model for stochastic programs with "chance constraints".

References: [30], [37], [61], [63], [8A], [9A], [10A], [11A], [13A], [19A], [24A], [42A], [46A]

Alternates: David W. Walkup and Roger J. B. Wets, "Stochastic Programs with Recourse: Special Forms," Proceedings of the Sixth International Symposium on Mathematical Programming, H. W. Kuhn, ed., Princeton University Press, Princeton, New Jersey.

Referenced by: [24A], [28A], [36A], [46A]

[26A] A. Charnes, M. J. L. Kirby, and W. M. Raike, "Solution Theorems in Probabilistic Programming: A Linear Programming Approach," Journal of Mathematical Analysis and Applications, Vol. 20, No. 3, December, 1967, pp. 565-582.

The avowed purpose of this article is to correlate the results of other researchers in the subject area in an effort to present a new viewpoint on some of the previously studied problems. To this end several specific topics are studied in some detail. After a brief statement of the two-stage linear programming under uncertainty problem, three theorems are presented which produce alternate formulations in terms of a deterministic "constrained hypermedian" problem requiring consideration of only the first-stage decision variables. Each result arises from a specialization of the general two-stage problem.

The next section considers in some detail the case of two-stage linear programming under uncertainty in which the extremum of the objective functional is not actually attained but comes arbitrarily close to some limiting value (the "insoluable finite" case). The concern of the rather lengthy discussion presented is to establish the equivalence in limiting behavior of the objective functional under a known sequence of defining decision variables and another sequence which moves first to a finite extreme point of the constraint set and subsequently along an extreme edge of that convex polytope. The last paragraphs establish an asymptotic formula for the radial directional derivative of the objective functional under the assumptions imposed, an expression for the gradient of the objective functional given a continuously differentiable distribution function for the problem random variable, and an expression for the matrix of second partial derivatives of the objective form.

The last topic considered is the general N-stage linear programming under uncertainty problem. An alternate proof of a previous result is supplied and some observations as to computational procedures for solution of this class of problems is made.

References: [10], [50], [8A], [9A], [10A], [11A], [12A], [15A], [24A], [35A], [42A].

Alternates: A. Charnes, M. J. L. Kirby, and W. M. Raike, "On a Special Class of Constrained Generalized Median Problems," The Technological Institute, Northwestern University, Evanston, Ill., Systems Research Memorandum No. 155, April, 1966.

Referenced by: [9A], [21A].

[27A] W. T. Ziemba, "On Nonlinear Stochastic Programming with Simple Recourse," Center for Research in Management Science, University of California, Berkeley, Working Paper No. 237, December, 1967.

This paper reviews the literature on two-stage stochastic programs through 1967. Although some mention is made of the "general" two-stage stochastic program formulations available in the literature, the discussion in the main is concerned with the "complete" or "simple recourse" problem. Two decision problems (an inventory situation and an economic system model) are described and cast as two-stage programming under uncertainty problems. Sufficient conditions to insure continuous differentiability of the objective function of the equivalent convex program are given. These are illustrated for the case of a separable objective incorporating quadratic terms in the second-stage decision variables by presenting the objective function and its derivatives for the cases of normally, exponentially, uniformly, and multivariate normally distributed right-hand-side requirements vector variation laws. In the last instance a detailed reduction of the expressions is not given.

References: [10], [37], [39], [2A], [4A], [8A], [10A], [11A], [13A], [20A], [24A], [42A].

Alternates: William Thomas Ziemba, "Essays on Stochastic Programming and the Theory of Economic Policy," Ph.D. Thesis, Graduate School of Business Administration, University of California, Berkeley, June 1969. W. T. Ziemba, "Nonlinear Stochastic Programming [illegible] le Recourse," Faculty of Commerce and Business [illegible] tion, University of British Columbia, Working Paper No [illegible] pril, 1969.

Referenced by: [44A], [47A].

[28A] Richard M. Van Slyke and Roger J.-B. Wets, "Stochastic Programs in Abstract Spaces," Stochastic Optimization and Control (Proceedings of an Advanced Seminar Conducted by the Mathematics Research Center and the United States Army at the University of Wisconsin, Madison, October 2-4, 1967), Herman F. Karreman, ed., John Wiley & Sons, Inc., New York, 1968, pp. 25-45.

This work purports to extend the existing theory relative to two-stage stochastic linear programs defined in finite dimensional spaces to the case of more general abstract spaces. This is done by considering reflexive Banach spaces, separable probability spaces, linear operators, and convex functionals in the restatement of the usual formulation which uses n-dimensional Euclidean space and matrices as linear operators.

A portion of the work is related to the discussion of stochastic control problems as two-stage stochastic linear programs. Here some well-known results for stochastic control problems which can be cast in terms of linear vector differential equations are stated and discussed relative to possible stochastic variation in the equation parameters. In the discussion a relationship to the two-stage stochastic programming problem is established.

Another, larger portion of the paper is taken up with the equivalent deterministic problem and the set of feasible solutions for the subject "generalized" two-stage linear programming problem. The discussion of these sections is taken up with a restatement of several Theorems and proofs of the previous papers cited in terms of the more abstract formulation defined in the opening remarks to the paper. A final section remarks on the connection of the previous discussion to stochastic programs which are completely analogous with respect to the linearity and regularity conditions imposed to the usual linear programming formulation.

References: [9], [15], [37], [1A], [2A], [9A], [10A], [11A], [12A], [13A], [18A], [24A], [25A].

Alternates: Richard M. Van Slyke and Roger J.-B. Wets, "Stochastic Programs in Abstract Spaces," Boeing Scientific Research Laboratories, Document D1-82-0672 (Mathematics Research Laboratories, Mathematical Note No. 539), October, 1967.

Referenced by:

[29A] A. C. Williams and M. Avriel, "Remarks on 'Linear Programming Under Uncertainty'," Operations Research, Vol. 16, No. 1, January-February, 1968, pp. 198-203.

This note is a brief remark on an earlier article which presented a dual program to the two-stage linear programming under uncertainty problem in the case of a joint normal distribution characterizing the components of the stochastic right-hand-side vector. A dual problem suitable for two-stage linear programming under uncertainty problems with arbitrary continuous probability distribution functions with finite first moments is stated. It is shown that specialization of this result produces the form of the previous paper. It is further established that the optimal solution does not explicitly depend on the mean and variance of the marginal distributions of the random right-hand-side coefficient variables.

References: [8A], [9A], [10A], [11A], [13A], [14A].

Alternates:

Referenced by: [21A], [47A].

[30A] M. A. H. Dempster, "On Stochastic Linear Programming I. Static Linear Programming Under Risk," Journal of Mathematical Analysis and Applications, Vol. 21, No. 2, February, 1968, pp. 304-343.

This paper represents the single most comprehensive study of "two-stage linear programming under uncertainty" and its varients available in the literature to the date of the article. The approach adopted in the main is geometric rather than algebraic. That is, the results presented are described in terms of the theory of convex polyhedra with frequent appeal to geometric constructions rather than proceeding on the basis of algebraic manipulation of the basic problem formulation. A must article for the serious student of probabilistic programming.

The paper is divided into six sections. Section 1 is introductory and describes the problem to be presented in general terms by relating it to the appropriate analogous discussions in the available open literature. Section 2 is brief and presents the notation and definitions to be adopted throughout the remainder of the text. Section 3 presents a formulation of "the discrepancy cost approach to stochstic programming" and indicates its place in respect to the general area of mathematical programming and the "chance constrained" and "stochastic linear programming" approaches to problems of probabilistic programming. The "loss function" of the "discrepancy cost" approach is introduced and discussed in its general abstract linear form. Section 4 introduces the concept of "quadratic loss" and very briefly considers its effect on the fundamental problem to be considered.

Sections 5, 6, 7, and 8 comprise the "meat" of this article. Section 5, through its discussion of convex polytopic cones, linear inequalities, and pseudo-inverses (called elsewhere "Morse-Penrose generalized inverses"), suffices to characterize completely the general nature of the second-stage problem constraint set. Proofs and interpretations of several Theorems including those of Minkowski-Weyl and Minkowski-Farkas are given and related to the problem at hand. Section 6 continues the development along the same lines by establishing necessary and sufficient conditions for the total discrepancy cost program to have a feasible solution. It indicates some optimality conditions which the optimal feasible solution must satisfy. Section 7 discusses discrepancy cost stochastic programs in which the second-stage constraint set establishes the set of feasible solution vectors as a pointed cone. Deterministic equivalents for the stochastic program are developed and the constrained linear nature of the program objective functional exhibited. Section 8 performs a similar service for the case in which the set of feasible solution vectors is a linear subspace. In this case the program is reducible to the "complete" formulation and the loss function exhibits piecewise linear behavior in the program first-stage decision variables. The concluding remarks of this section suggest that the techniques suggested for the problem dichotomy established in Sections 7 and 8 provide the theoretical basis for the description and solution of the general "stochastic program with linear discrepancy cost" or "two-stage linear programming under uncertainty" problem.

References: [5], [6], [7], [9], [10], [20], [26], [27], [30], [37], [38], [43], [45], [50], [53], [54], [57], [58], [8A], [11A], [24A].

Alternates: Michael Alan Howarth Dempster, "On Stochastic Programming," Ph.D. Thesis, Carnegie Institute of Technology, College of Engineering and Science, May, 1965.

Referenced by: [22A], [46A].

[31A] David Rutenberg, "Risk Aversion in Stochastic Programming with Recourse," Management Sciences Research Group, Graduate School of Industrial Administration, Carnegie Mellon University, Management Sciences Research Report No. 122, February 1968

One previous two-stage linear programming under uncertainty problem which has appeared in the literature proposes the solution of problems with discretely distributed right-hand-side random coefficients through consideration of the large linear program obtained by writing all possible program occurrences in a single tableau. "Relative cost coefficients" for the program are defined as those occurring in the parent formulation weighted by the probability of occurrance of the right-hand-side vector event corresponding to each "second-stage" problem generated. The program objective is to extremize the expectation (here a finite sum of positively weighted linear terms) of the associated criterion function.

The subject paper uses the above formulation as a point of departure to question the general applicability of assuming a utility function which is linear in the program decision variables in situations of decision making under risk. It is argued that a more reasonable approach would be to adopt a criterion function which will reflect the "risk aversion" behavior commonly exhibited by decision makers when faced by uncertainties, i.e., a criterion function concave in the program decision variables should be considered. Such a function is incorporated into the formulation in terms of its general characteristics and a solution approach based upon two well-known contemporary convex programming solution algorithms is outlined. Suggestions as to avenues for further research efforts are made in the concluding remarks.

References: [10], [37], [8A], [10A], [13A], [24A], [42A].

Alternates:

Referenced by:

[32A] R. S. Sachan, "On the Nonlinear Stochastic Programming Problems," Cahiers du Centre d'Études de Recherche Opérationelle, Vol. 10, No. 2, 1968, pp. 84-99

This paper appears to be a restatement of the results obtained by other authors in some previous papers. The extension arises in that the right-hand-side random variables and the objective function coefficients are allowed to be functions of one another. Inequalities between the "expected value", the "fat", and the expected a posteriori solutions objective function return realized are developed (among others). Some simple illustrative numerical examples are presented in the final section.

References: [1A], [2A], [3A], [10A], [16A], [24A].

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[33A] Nak Je Kim, "Linear Programming with Random Requirements," report Utah Water Research Laboratory, Utah State University, Logan, Utah, 1968

Not received for review Indications are that this is a very simple survey article.

References:

Alternates: This entry was located by reference to a preliminary version of a "Bibliography on Programming Under Risk," supplied by Mr. Douglas V Smith, Harvard University Center for Population Studies, 22 Plympton Street, Cambridge, Massachusetts 02138. He indicates that it describes "Some simple examples bearing no relation to water resource problems "

Referenced by.

[34A] J. L. Midler and R. D. Wollmer, "A Flight Planning Model for the Military Airlift Command," The RAND Corporation, Santa Monica, California, Memorandum RM-5722-PR, October, 1968.

This paper is the first of a series to be concerned with the development and implementation of a flight scheduling model for the Military Airlift Command. The basic problem is one of scheduling the allocation of aircraft to routes so that anticipated costs, including those arising as a result of uncertain transport requirements, are minimized. A monthly planning model is used to develop an initial schedule and a daily model operates within the monthly period to define needed schedule adjustments and other compensatory activities as the state of the system evolves. Both models are developed as two-stage linear programming under uncertainty problems having "complete" recourse.

The total problem, as is noted, is in actuality a multiple period sequential decision problem. The formulation given incorporates a two-level, nested set of two-stage programs with "virtual" or fictitious second-stage decisions in order to produce a computationally feasible algorithm. Each monthly schedule is generated on the basis of the current status of the system and a second-stage decision which accounts for uncertainty in the forthcoming month viewed as a single decision period. This second-stage decision is never realized in that the projected schedule is input into the daily model to be operated upon by the daily model as the month progresses. The daily model, operating recursively on a two stage basis (outputs on a given day are inputs for the next), produces a revised statement of the state of the system at month's end which is used in determining the next month's schedule projection and so on.

Two computational methods applicable to problems with continuously distributed random right-hand-side vectors are outlined (a cutting plane method and a gradient minimization method). In this discussion a good introductory statement of the theoretical aspects of constraint generation and the two proposed solution techniques is made. A small-scale numerical example of the solution of the monthly model comprises the final section of the paper.

References: [37], [10A], [11A], [35A], [42A]

Alternates:

Referenced by:

[35A] Katta G. Murty, "Two-Stage Linear Programming Under Uncertainty: A Basic Property of the Optimal Solution," Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete, Vol. 10, No. 4, October, 1968, pp 284-288

If a finite optimal solution to a two-stage uncertain linear program exists, the main result of this paper assures that the solution is describable in terms of a set of linearly independent column vectors which correspond to the positive components of the first-stage decision variables appearing in the optimal solution. A bound on the number of column vectors is also given. It is noted that the stated result is easily extended to the case of an analogously formulated multi-stage linear programming under uncertainty problem.

References: [10], [37], [63], [10A], [15A].

Alternates: K. G. Murty, "Two Stage Linear Programming Under Uncertainty: A Basic Property of the Optimal Solution," Operations Research Center, University of California, Berkeley, ORC 66-4, February, 1964.

Referenced by: [26A], [34A], [42A].

[36A] David W. Walkup and Roger J.-B. Wets, "A Note on Decision Rules for Stochastic Programs," Journal of Computer and System Sciences, Vol. 2, No. 3, October, 1968, pp. 305-311.

The purpose of this paper is to demonstrate the fact that the objective form of the usual two-stage linear programming under uncertainty problem is piecewise linear in the first-stage decision variables. It is also demonstrated, by means of a simple example, that the same behavior is not exhibited by the objective forms of problems involving three (or more) distinct decision stages. The implications of the conclusion reached with respect to some of the better known contemporary sequential probabilistic programming models is noted also.

References: [10], [10A], [11A], [24A], [25A], [40A]

Alternates: David W. Walkup and Roger J. B. Wets, "A Note on Decision Rules for Stochastic Programs," Boeing Scientific Research Laboratories, Document D1-82-0727 (Mathematics Research Laboratories, Mathematical Note No 560), May, 1968.

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[37A] R. Jagannathn, "A Solution Procedure for a Class of Multistage Linear Programming Problems Under Uncertainty," Management Sciences Research Group, Graduate School of Industrial Administration, Carnegie-Mellon University, Management Sciences Research Report No. 146, November, 1968.

The tone of this article is set by the opening remarks in which the multistage linear programming under uncertainty problem is described by a simple dynamic programming formulation. It is shown that this description is appropriate as an inventory model description and under suitable assumptions yields a convex programming problem. A solution algorithm for the multistage program for the case of a single stochastic constraint per decision period is discussed. The procedure developed is based upon approximating the possibly unseparable objective form for each decision stage by a separable function obtained through estimating the "certainty equivalents" for the program random variables. The net result is a series of convex programs each in a single decision variable instead of a single multistage problem in several decision variables. The extension to the case of several stochastic linear constraints per period then follows quite easily. A possibility for application of the final model is exhibited by formulating a capital budgeting decision process in terms of the mathematical structure developed earlier in the paper. No numerical results are given.

References: [63], [8A], [22A], [24A], [42A].

Alternates: Rajagopalan Jagannathn, "Solution Procedures for Multi-Stage Linear Programming Under Uncertainty and Applications," Management Sciences Research Group, Graduate School of Industrial Administration, Carnegie-Mellon University, Management Sciences Research Report No. 158, May, 1969 (Ph.D. Thesis).

Referenced by:

[38A] Gifford H. Symonds, "Chance-Constrained Equivalents of Some Stochastic Programming Problems," Operations Research, Vol. 16, No. 6, November-December, 1968, pp. 1152-1159.

In this paper it is shown that, given a strictly increasing, continuous, differentiable set of marginal density functions for the right-hand-side random variables of a two-stage linear program under uncertainty, an equivalent chance constrained programming problem can be exhibited. In the main the results achieved rest on the optimality conditions of appropriate Lagrangian saddle-point functions. The final result extends the rationale to the case in which the contribution of the second-stage decision variables produces a nonlinear objective function which is separable and convex in the program decision variables.

References: [10], [20], [26], [27], [3?], [41], [50], [63], [6A], [9A], [12A], [13A].

Alternates:

Referenced by:

[39A] David P. Rutenberg and Stanley J. Gartska, "Computation in Discrete Stochastic Programs with Recourse," Management Sciences Research Group, Graduate School of Industrial Administration, Carnegie-Mellon University, Management Sciences Research Report No. 151, January, 1969.

The problem of developing a computational scheme for stochastic programs with recourse (two-stage linear programs under uncertainty) when the right-hand-side constraint vector is described by a discretely distributed random variable is the subject of this paper. The procedure discussed involves systematic enumeration of all of the possible occurances of the right-hand-side vector via a branch and bound technique. Each of the possible basis matrices for the second-stage decision variable constraint matrix is investigated relative to the right-hand-side vector occurances for which it guarantees a feasible second-stage decision. Available techniques from the theory of parametric linear programming are used in the assessment. Although not explicitly pointed out, the elimination could be used to determine if a feasible solution to the total program exists and if so, to generate additional constraints for imposition on the first-stage decision variables which guarantee second-stage feasibility for decisions drawn from the resultant first-stage decision variable constraint set.

References: [37], [24A], [42A]

Alternates:

Referenced by:

[40A] David W. Walkup and Roger J.-B. Wets, "Stochastic Programs with Recourse II: On the Continuity of the Objective," Journal of the Society of Industrial and Applied Mathematics, Vol. 17, No. 1, January, 1969, pp. 98-103

The main result of this note is to establish that the expectation of a continuous convex function which incorporates a random variable as one of its arguments is again a continuous convex function. This of course requires that the expectation be lower semicontinuous over its range. Following this result an analogous Theorem appropriate to the description of an objective form for a stochastic program with recourse is stated. An example is given which is illustrative of the reasons that in general, one may not presume that the expectation of a convex function is continuous.

The discussion of the primary thought of this paper is self-contained. It is unfortunate, perhaps, that the subsequent example draws upon notation and ideas presented in a previous paper.

References: [24A].

Alternates: David W. Walkup and Roger J.-B. Wets, "Stochastic Programs with Recourse II: On the Continuity of the Objective," Boeing Scientific Research Laboratories, Document D1-82-0704 (Mathematics Research Laboratory, Mathematical Note No. 548), January, 1968.

Referenced by: [36A], [45A], [46A].

[41A] A. C. Williams and M. Avriel, "The Evaluation of Information for Stochastic Linear Programming," title of a paper presented at the 16th International Meeting of The Institute of Management Sciences, New York, New York, March 26-28, 1969

Not received for review

References.

Alternates:

Referenced by. [46A]

[42A] R. M. Van Slyke and Roger Wets, "L-Shaped Linear Programs with Applications to Optimal Control and Stochastic Programming," Journal of the Society of Industrial and Applied Mathematics, Vol. 17, No. 4, July, 1969

An L-shaped linear program is a linear program in which the nature of the set of linear constraints is such that (by suitable rearrangement, if necessary) the non-zero entries of the coefficient matrix corresponding to the program decision variables exhibit a "block L" structure. This form is exploited in this paper by partitioning the program constraints and decision variables into the two natural subsets defined. A two-stage primal-dual solution procedure is proposed which turns out to be a cutting plane generating algorithm. The duality of this approach with the well known decomposition method of solution for large scale linear programs is noted.

Although no computational results are presented, separate discussions on the method's applicability to solution of optimal control problems with state constraints and two-stage uncertain linear programs are given. The geometric interpretations with respect to convex polyhedral cones and sets interspersed throughout the text aid the presentation.

References: [37], [63], [10A], [11A], [15A], [35A], [46A].

Alternates: R. M. Van Slyke and Roger Wets, "L-Shaped Linear Programs with Applications to Stochastic Optimal Control and Stochastic Programming," Boeing Scientific Research Laboratories, Document D1-82-0526 (Mathematics Research Laboratories, Mathematical Note No. 520), August, 1967. Also by the same authors and under the same title as ORC 66-17, Operations Research Center, University of California, Berkeley, July, 1966.

Referenced by: [20A], [25A], [28A], [31A], [34A], [37A], [39A], [46A].

[43A] R. N. Braswell and F. M. Allen, "Tolerance Limits for an Uncertain Requirements Vector in LP," preprint of a paper presented at the 1969 Joint National Meeting of the Operations Research Society of America and the American Astronautical Society, Denver, Colorado, June 17-20, 1969

This paper considers linear programming problems in which the right-hand-side vectors are known to be random variables but for which the distributions are unknown. The questions which arise are therefore to be discussed in terms of considerations of distribution-free statistical studies. It is assumed that one is required to initiate activity upon the basis of crude estimates for the system stochastic parameters and subsequent modifications to the original plan are allowable. The question now arises as to the advisability of changing the implemented solution once observations on the system random parameters have been taken. This decision is made with reference to the cost of making the changes. These might involve changing the level of the activities of the original program or revising the set of optimal decision activities (as parametric linear programming analysis will demonstrate). It is argued that hypothesis tests based upon distribution-free statistical analysis of the observations can be used to make a more meaningful decision with respect to the significance of the variations observed than a mere ad hoc re-examination of the former solution. Wilk's method and a confidence band approach are given in outline form as examples of appropriate tests.

References: [5], [10], [26], [37].

Alternates: Paper title in ORSA/AAS Program: "On Non-Parametric Sensitivity in Linear Programming." Also, Robert N. Braswell and Frederick M. Allen, "Distribution-Free Requirements Vector, b, in Linear Programming," Department of Industrial and Systems Engineering, University of Florida, Gainesville, Florida, Report No. THEMIS-UF-TR-11 AROD-T-1:23-RT, January, 1969.

Referenced by:

[44A] William T. Ziemba, "Solving Non Linear Programming Problems with Stochastic Objective Functions," Faculty of Commerce and Business Administration, The University of British Columbia, Vancouver, British Columbia, Working Paper No. 42, July, 1969.

This paper discusses the possibility of using the interior parametric sequential unconstrained maximization technique (SUMT) for solution of nonlinear programming problems with special reference to two-stage probabilistic programming problems. An outline for the SUMT procedure and the assumptions necessary for its applicability to a particular problem is given The merits of this approach relative to the solution of two-stage stochastic programs with simple recourse are then indicated. A portfolio selection problem is formulated and cast into a form suitable for solution through the application of SUMT

References: [14] [27], [28], [34], [1A] [4A], [11A], [13A], [18A], [27A].

Alternates:

Referenced by:

[45A] David W. Walkup and Roger J.-B. Wets, "Some Practical Regularity Conditions for Nonlinear Programs," Journal of the Society of Industrial and Applied Mathematics on Control, Vol. 7, No. 3, August, 1969.

A nonlinear program is solvable if the optimal objective function value is finite and achieved for an admissible decision variable; it is dualizable if the primal and dual optimal solutions are equal; and it is stable if an optimal solution can be achieved through use of the Lagrangian formulation. This note establishes sufficient conditions for a class of nonlinear programs to possess these properties. In particular, it describes that class which reflects the inherent structure of the equivalent convex program of two-stage linear programming under uncertainty problems.

References: [10A], [11A], [24A], [40A].

Alternates: David W. Walkup and Roger J.-B. Wets, "Some Practical Regularity Conditions for Nonlinear Programs," Boeing Scientific Research Laboratories, Document D1-82-0792 (Mathematics Research Laboratory, Mathematical Note No. 583), November, 1968.

Referenced by:

[46A] Roger J.-B. Wets, "Stochastic Programs with Recourse: A Survey I.," Boeing Scientific Research Laboratories, Document D1-82 0882 (Mathematics Research Laboratory, Mathematical Note No.), August, 1969.

This paper is divided into eight sections. Sections 1, 2, and 3 are introductory and are concerned with describing the relationship of stochastic programming problems to the theory of decision making under "risk" and "uncertainty", with a formal statement of the problem of "stochastic programming with fixed recourse", and with defining an equivalent deterministic program respectively. The topic of Section 4 is the set of feasible solutions for the subject program. A constructive definition of the feasibility region is given and conditions necessary for the set of feasible solutions to be a convex polyhedral set (or a convex set) are discussed. Of particular import are feasibility regions whose lower bounds are obtainable by a coordinate-wise investigation of the bounds of the constituents of the decision vector and which in fact contain the lower bound defined through juxtaposition of the coordinate bounds. The procedures suggested in Section 5 for defining a set of feasible solutions for stochastic programs with fixed recourse are greatly simplified if such a bound exist. In particular, the cutting plane algorithm presented reduces to a relatively simple linear programming problem if, in addition, the set of points in the support of the program random variable is polyhedral. Sufficient conditions for the existence of such a lower bound are also discussed.

Section 6 is short and defines stochastic programs with fixed recourse in which all vectors feasible for the first-stage decision are also feasible with respect to the second-stage problem. It is noted here that the problem of ascertaining feasibility is one of determining if a given convex set is a subset of a given convex polyhedral cone.

The objective function of the equivalent deterministic program is investigated in Section 7. It is shown to be continuous, convex, and Lipschitz, and, under appropriate conditions, to be polyhedral and piecewise linear in the first-stage decision variables. Several related inequalities are developed which could be used as a basis for solution procedures to define the extremizing solution vector. These lead quite naturally to the brief discussion of Section 8 on the existence of primal-dual pairs of deterministic equivalent programs for stochastic programs with fixed recourse.

This article is a survey and for that reason does assume statements the proofs of which are left to the cited references. Even so, the discussion for the most part is self-contained and comprehensive. The geometrical mode of presentation provides an intuitive basis for the mathematical statements given.

References: [9], [10], [15], [27], [30], [37], [59], [60], [63], [8A], [9A], [10A], [11A], [15A], [18A], [20A], [24A], [25A], [30A], [36A], [40A], [41A], [42A].

Alternates: This paper incorporates material referenced elsewhere under the titles "Stochastic Programs with Recourse: Addendum," and "Finding a Feasible Solution to Stochastic Programs with Fixed Recourse."

Referenced by: [25A], [42A].

[47A] W. T. Ziemba, "Duality Relations, Certainty Equivalents and Bounds for Convex Stochastic Programs with Simple Recourse," Cahiers du Centre d'Études de Recherche Opérationelle.

This paper touches upon several closely related topics by correlation of the existent results of other researchers. A special class of two-stage uncertain linear programs in which the second-stage decision is uniquely determined as a result of the first-stage decision and observation of the random right-hand-side vector of the second-stage constraints is considered. In addition, it is assumed that the convex objective form associated with the problem has continuous second partial derivatives with respect to the program decision variables.

Under the given assumptions it is possible to apply an existing duality theorem to produce a series of deterministic problems which are equivalent to the original stochastic formulation. Given these necessary and sufficient conditions for the existence of a "certainty equivalent" problem for the original formulation and described and upper and lower bounds on the program objective functional established. An interesting economic interpretation of the primal and dual objective forms is followed by an illustrative numerical example to conclude the paper.

References: [10], [1A], [2A], [9A], [11A], [13A], [14A], [21A], [27A], [29A].

Alternates: W. T. Ziemba, "Duality Relations, Certainty Equivalents and Bounds for Convex Stochastic Programs with Simple Recourse," paper presented at the 1969 Joint National Meeting of the Operations Research Society of America and the American Astronautical Society, Denver, Colorado, June 17-20, 1969. Also, William Thomas Ziemba, "Essays on Stochastic Programming and the Theory of Economic Policy," Ph.D. Thesis, Graduate School of Business Administration, University of California, Berkeley, June 1969.

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APPENDIX I

AUTHOR INDEX TO LITERATURE ABSTRACTS

APPENDIX 1

AUTHOR INDEX TO LITERATURE ABSTRACTS

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[26A] __________, M J L Kirby, and W. M Raike, "Solution Theorems in Probabilistic Programming: A Linear Programming Approach," Journal of Mathematical Analysis and Applications, Vol. 20, No. 3, December, 1967, pp. 565-582.

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APPENDIX II

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13. ABSTRACT

This Bibliography is intended to summarize the available literature on "Two-Stage Linear Programming Under Uncertainty" for the period January, 1964 through August, 1969. Each of the 47 entries presents a summary of the principal thoughts of the article reviewed, headed by a complete bibliographic reference based upon the most up-to-date information available. Alternate sources are quoted as a part of each entry. All entries are cross indexed by author and title relative to the primary chronological order sequence.

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